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Abstract

Breakdown waves propagating in the opposite direction of the applied electric field force are referred to as antiforce waves. Breakdown waves moving into a pre-ionized medium are referred to as Class II waves. Using a one-dimensional, steadystate, three-fluid, hydrodynamical model and considering the electrons as the main element in propagation of ionizing waves, we have derived the proper boundary conditions for antiforce waves moving into a preionized medium. Using the new boundary conditions and for several current values ahead of the wave, the set of electron fluid dynamical equations (equations of conservation of mass, momentum, and energy coupled with Poison's equation) has been integrated through the dynamical transition region. The solutions meet the expected boundary conditions at the end of the wave. The electron velocity and electric field values conform to the physical conditions at the end of the dynamical transition region. For several current values ahead of the wave, the wave profile for electric field, electron velocity, electron temperature, and electron number density will be presented.

Introduction

All attempts of using Maxwell's equations to explain breakdown waves have failed; even though this phenomenon has been known for some time, it is wide open for explanation. The earliest person known to pay attention to these waves of luminous pulses in an evacuated chamber was Hauksbee (1706/7). During the last several decades, many experiments have been conducted, but the data recorded have been incomparable. The individual experiments were done with different procedures; there was not a standard procedure. For example, experimenters used different geometrical configurations for the gas chambers and used different gases in their chambers.

J. W. Beams (1930) did experiments on the electrical gas discharge in air and hydrogen. He recorded that breakdown waves travel from the electrode with the applied potential toward the other electrode that is grounded, disregarding the polarity of the potential applied. L. B. Loeb (1965) did studies of luminosity of the spark breakdown in which he suggested that potential waves are present during this phenomenon. Loeb first proposed a qualitative model for the propagation of a streamer, as a mechanism for electrical breakdown in a gas. The mechanism of the streamer process proposed by Loeb for a point-anode plane-cathode geometry is as follows: "Photons emitted by the excited gas molecules diffuse outward from the anode, ionizing and exciting new molecules. The photons emitted by the newly excited molecules diffuse further into the gas, and the cycle of diffusion, excitation, ionization and photon emission is repeated. The net result of this process is the propagation of a photoionization wave from the anode to cathode."

Beams (1930) provided a qualitative explanation of the pulse. He said that the gas behind the wave is electrically conducting. This means that the wave is carrying the potential from the charged electrode and discharging it to the electrode that is grounded. A high electric field occurs at the front of the wave, and the motion of the wave front is due to the mass difference of the electrons and the positive ions. Larger diameters of the tube and a higher potential difference on electrodes increases the velocity of the wave front. Higher initial pressure also increases the velocity until a pressure of a few Torr, and then the velocity begins to decrease.

It is presumed that the ionization of a small amount of the gas occurs where the potential gradient is greatest. The electrons produced receive their kinetic energy from the electric field. The condensed, high-temperature electron gas expands, creating a shock wave, which propagates down the tube into the neutral or undisturbed gas partially ionizing the gas molecules. Energy given by the external electric field drives the shock wave. The shock wave is followed by another wave known as the rarefaction wave.

Electron driven shock waves moving into a non-ionized medium are called Class I waves. Shock waves moving into an ionized medium have a different structure than Class I waves. Waves that move in a medium of significant electron density are known as Class II waves. For example, this would occur when a Class I wave has already moved through the medium leaving a high electron concentration behind. After an initial lightning discharge has occurred in the atmosphere, the medium left behind is composed of positive nitrogen ions and negative oxygen ions, with equal number densities.

Far ahead of the wave front, a weak electric field ($E\infty$) accelerates the positive and negative ions to a speed of v_o in opposite directions, which creates a current ahead of the wave front. The current ahead of the wave will alter the shock conditions for electron temperature as well as Poisson's equation.

The equations of conservation of mass, momentum, and energy for Class II Waves will remain the same as the ones for Class I waves (Hemmati et al., 1999). In dimensionless variables the equations respectively are

$$\frac{d}{d\xi}(\nu\psi) = \kappa\mu\nu, \tag{1}$$

$$\frac{d}{d\xi}[\nu\psi(\psi-1)+\alpha\nu\theta] = -\nu\eta-\kappa\nu(\psi-1). \tag{2}$$

$$\frac{d}{d\xi}[\nu\psi(\psi^2-1)+\alpha\nu\theta(5\psi-2)+\alpha\nu\psi+\alpha\eta^2-\frac{5\alpha^2\nu\theta}{\kappa}\frac{d\theta}{d\xi}]$$
$$= -\omega\kappa\nu[3\alpha\theta+(\psi-1)^2]. \tag{3}$$

The dimensionless variables are

$$\omega=\frac{2m}{M}, \quad k=(\frac{mV}{eE_o})K, \quad \mu=\frac{\beta}{K}, \quad \alpha=\frac{2e\phi_1}{mV^2}, \quad \eta=\frac{\epsilon_o E_o^2}{2e\phi}\upsilon,$$

$$T_e=(\frac{2e\phi}{k})\theta, \quad E=\eta E_o, \quad x=-(\frac{mV^2}{eE_o})\xi, \quad v=V\Psi.$$

In the above equations ν, ψ, θ, μ, κ, η, and ξ are the dimensionless electron number density, electron velocity, electron temperature, ionization rate, elastic collision frequency, electric field, and position inside the wave, respectively. The symbols n and T_e represent electron number density and temperature inside sheath, and β, ϕ, V, M, and E_o are ionization frequency, ionization potential, wave velocity, neutral particle mass, and electric field at the wave front, respectively.

Solution of the Equations

Assuming that the wave is moving in the positive x direction with a velocity of V_o referenced to the lab frame, in the wave frame the neutral particles will be swept into the wave front at a velocity of $-V_o$. Ahead of the wave the positive and negative ions in the electric field E_∞ will enter the wave front with velocities of $-(V_o + v_o)$ and $-(V_o - v_o)$ or vice versa. Therefore, the equation of conservation of current at the wave front will become

$$env\text{-}eN_iV_o=en_o(V_o - v_o) - en_o(V_o + v_o). \tag{5}$$

When the wave front propagates into the ionized medium, the ions go from being under the influence of a weak electric field E_∞ to a strong electric field E_o. The electric field in the wave front will strip loosely bound electrons from the negative ions and drive the loose electrons away from the wave front with a speed v. It is assumed that the electron temperature and the electron gas pressure are large enough to drive the wave.

To find the initial condition on electron temperature, the global momentum equation is integrated and the constant of integration is evaluated by the conditions ahead of the wave. The global momentum equation reduces to

$$n_1mv_1^2 + N_{i1}(M-m)V_o^2 + MNV_o^2 + n_1kT_{e1}) + (N+N_i)kT +$$
$$\frac{\varepsilon_o}{2}(E_o^2 - E_o^2) - n_okT_{eo} - (N_o + N_{io})kT_o - n_om(V_o - v_o)^2 -$$
$$MN_oV_o^2 = 0. \tag{6}$$

The electron and neutral particle temperatures ahead of the wave are T_{eo} and T_o, respectively. Instead of $E\infty$, the electric field at the wave front is E_o. The particle densities $N = N_o$, and $N_i = N_{io} = n_i$ because the ionization occurs inside the sheath, not at the wave front. As a result, $(N+N_i)kT_o$ and $(N_o + N_{io})kT_o$ cancel each other. The electron temperature ahead of the wave is at room temperature, which becomes negligible compared to the electron temperature inside the sheath. The wave velocity, V_o, is much larger than the neutral-ion velocity, v_o, ahead of the wave. Therefore, the terms containing v_o^2 are neglected. Taking all of these assumptions and employing the equation of conservation of heavy particles further reduces the equation. Introducing the dimensionless variables, the initial condition on electron temperature becomes

$$\theta_1 = \frac{\psi_1}{\alpha}(1-\psi_1) + \frac{2\psi_o\nu_o}{\alpha\nu_1}(\frac{M}{m} - 1). \tag{7}$$

θ_1, ν_1, and ψ_1 are the electron temperature, density, and velocity at the wave front, respectively. Solving the equation of conservation of current for N_i and substituting it in Poisson's equation, and introducing dimensionless variables will reduce it to

$$\frac{d\eta}{d\xi} = \frac{\nu}{\alpha}(1-\psi) - \frac{2\nu_o\psi_o}{\alpha}, \tag{8}$$

where ν_o and ψ_o are the dimensionless ion density and the electron velocity ahead of the wave. $J_o = \psi_0\, \nu_0$ is the dimensionless current ahead of the wave.

Analysis

Using the singularity inherent in the equation set we were able to integrate the set of electron fluid dynamical equations by trial and error. The complete set of equations is composed of the equations of conservation of mass, momentum, and energy, eqs. (1) to (3) and Poisson's equation [eq. (8)]. The method of integration of the set of equations is given in an earlier publication (Hemmati et al., 1999). The following are the results of the integration of the equation set for several current values 0, 2 x 10^{-6}, 6 x 10^{-7}, and 6 x 10^{-8} ahead of the wave.

The electron velocity at the wave front is less than the wave velocity, therefore $\psi_1 < 1$. Fig. 1 shows a plot of the dimensionless electron velocity, ψ, as function of the electron position, ξ, inside the sheath. ψ reaches its maximum value at the middle of the sheath and finally reduces to one at the end of the sheath. This is the required condition at the end of the sheath because the electrons slow down to the same speeds as the neutral particles.

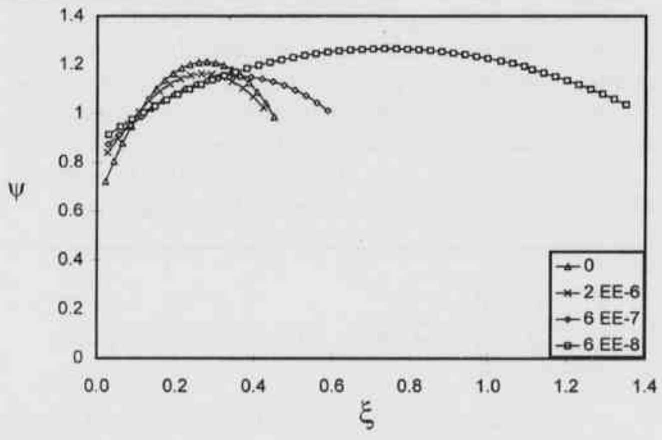

Fig. 1. Electron velocity, ψ, as a function of electron position, ξ, for four values of $J_o = 0$, 2 X 10^{-6}, 6 X 10^{-7}, and 6 X 10^{-8}.

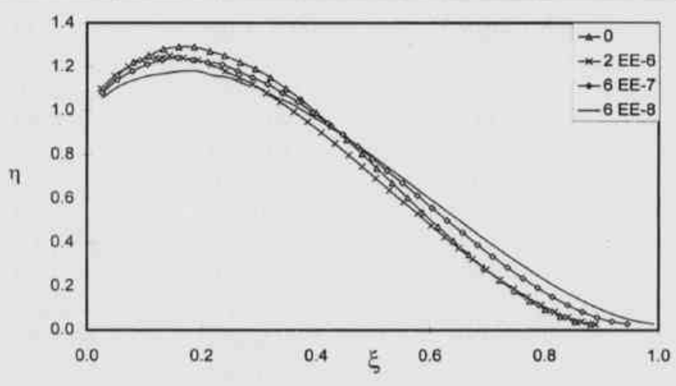

Fig. 2. Electric field, η, as a function of electron position, ξ, for four values of $J_o = 0$, 2 X 10^{-6}, 6 X 10^{-7}, and 6 X 10^{-8}.

The electric field, η, as a function of ξ, starts at 1 where $E = E_o$, and increases to reach a maximum; as expected, at the end of the sheath, the electric field reduces to zero. The ionized medium can not support an electric field, and the electric field variation in Fig. 2 agrees with the expected physical conditions.

Figs. 3 and 4 show the wave profile for dimensionless electron temperature, θ, and electron number density, ν, as a function of position, ξ, inside the sheath. Fig. 3 shows that the electron gas at the end of the sheath has considerable amount of thermal energy. This energy is utilized to further ionize the gas behind the sheath.

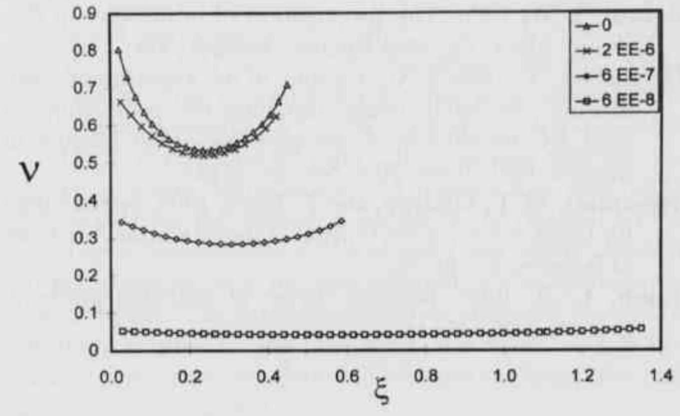

Fig. 3. Electron number density, υ, as a function of electron position, ξ, for four values of $J_o = 0$, 2 X 10^{-6}, 6 X 10^{-7}, and 6 X 10^{-8}.

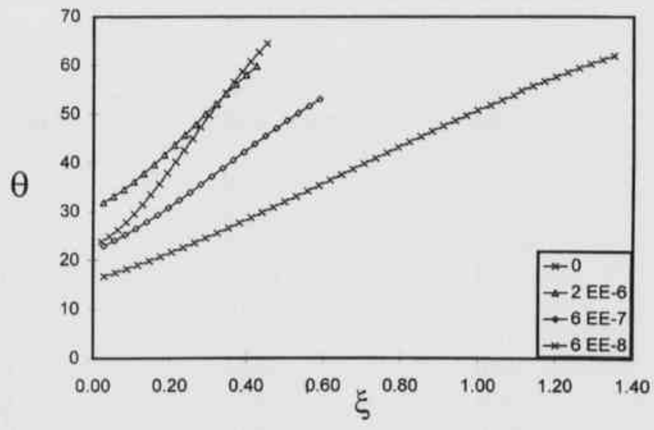

Fig. 4. Electron temperature, θ, as a function of electron position, ξ, for four values of $J_o = 0$, 2 X 10^{-6}, 6 X 10^{-7}, and 6 X 10^{-8}.

Conclusions

In conclusion, the electron fluid dynamical equations present a good model for the electrical breakdown waves propagating into a pre-ionized medium. The sheath thickness varies with different currents ahead of the wave. Also as expected, the electric field at the end of the sheath decreased to zero.

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